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Magnetic viscosity: outbursts and outflows in accretion driven systems

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Abstract. In this paper magnetic viscosity is investigated in magnetized accretion discs. It will be shown that the effective coupling between the magnetic field of a slow-rotator and an accretion disc, can be a very effective mechanism to drive episodes of high mass accretion onto the surface of a compact object. Outside the corotation radius, angular momentum is effectively transferred outwards through a propeller-type process from the magnetospheric field and magnetic bubbles that are formed as a result of a Kelvin-Helmholtz instability, which can result in a centrifugal barrier and accumulation of disc matter outside the corotation radius which will become unstable at some point, triggering enhanced inward mass advection as a result of a magneto-gravitational instability. This may lead to periods of enhanced mass accretion and associated disc brightening, which may explain the dwarf novae phenomenon in certain disc accreting cataclysmic variables. This may be accompanied by mass outflows from the disc and possible non-thermal emission. The description of magnetic viscosity presented in this paper will rely on the values of two constants, i.e. the Hartmann and Reynolds numbers of the magnetized disc plasma. For both these numbers above unity, magnetic stresses in the disc can play a very important role in the kinematics of the plasma in disc accreting systems.

Key words. accretion - binaries: general - stars: white dwarf - processes: accretion discs - turbulence: binaries: cataclysmic variables, dwarf novae

1. Introduction

Currently it is believed that most outbursts from disc accreting systems like the dwarf nova (DN) phenomenon are intimately linked to significant cyclic changes in the accretion disc viscosity. The quantification of accretion disc viscosity received enormous impetus with the incorporation of turbulent viscosity, which exceeds molecular viscosity by several orders of magnitude. In the seminal paper of Shakura & Sunyaev (1973), the turbulent viscosity is parameterized by $\nu_{\rm turb} = \alpha c_{\rm s} H$, where $\alpha < 1$ is a dimensionless parameter which scales the magnitude of the viscosity in terms of the sound speed $c_{\rm s}$ and the disc scale height H. Following this model, the hysteresis S-curve characterizing a DN outburst (Meyer & Meyer-Hofmeister 1981) in the $\Sigma - T_{\rm disc}$ plane has been quantified in terms of a cyclic α parameter variation. Simulated DN lightcurves resemble observed lightcurves most closely for values of $\alpha_{\rm cold} \sim 0.01$ and $\alpha_{\rm hot} \sim 0.1 - 0.3$ (Ichikawa & Osaki 1992). Although the introduction of turbulent viscosity through the α parameter proved to be a useful way of constraining the magnitude range of the disc viscosity, the absence of a detailed theory of turbulence means that it is no more than a neat way of hiding numerous ill-understood underlying principles. However, through a series of papers, it has been shown that magnetized discs are unstable against the magnetorotational instability (MRI) (Velikhov 1959). It has been shown (e.g. Balbus & Hawley 1991, as well as Balbus & Hawley 1998 and Balbus 2003 for reviews) that the strong coupling between the disc plasma and weak disc magnetic fields via the Lorentz force results in the MRI being an effective mechanism for angular momentum transfer in accretion discs. This mechanism also provides a more quantitative description of the viscosity variations between the "hot" and "cold" states of the accretion disc during the DN outbursts.

In this study, a different approach is followed. The effect of magnetic viscosity will be investigated in terms of the magnetic transport through the disc, which is described effectively by two parameters, i.e. the magnetic Hartmann (M) and Reynolds ($R_{\rm m}$) numbers (Jackson 1975). The Hartmann number is the ratio of magnetic stresses to particle viscosity in a magnetized fluid, while the Reynolds number essentially represents the ratio of the convection and diffusion associated with the magnetic field in the fluid. These two numbers are given respectively by

$$M = \left(\frac{\sigma B^2 L^2}{\mu c^2}\right)^{1/2} \tag{1}$$

$$R_{\rm m} = \left(\frac{4\pi L v \sigma}{c^2}\right), \tag{2}$$

where σ, μ, B, L and v represent the fluid electrical conductivity, the coefficient of dynamical viscosity, the magnetic intensity, the length scale of the field and the bulk flow speed respectively. For both the Hartmann and Reynolds numbers exceeding unity, the magnetic field will play an extremely important role in the dynamics of a magnetized fluid, which can have important consequences in astrophysical environments. In this study, the effect of magnetic viscosity will be investigated in disc accreting cataclysmic variables. It will be shown that magnetized discs can naturally explain episodes of transient accretion which are associated with the dwarf nova phenomenon. The paper will be structured as follows: In the next section a brief discussion will be presented of the role magnetic fields will play in slow rotators, followed by a discussion related to magnetic viscosity and the magnetogravitational instability, turbulent heating, outflows and a conclusion.

2. Magnetic fields and slow rotators

It has been shown (Campbell 1997) that for slow rotators with magnetic diffusivity a function of radial distance, the associated disc inflow speed can be determined, which is

$$-v_{\rm R} \propto \left(\frac{B_*^2 R_*^6}{M_* \dot{M}^2}\right)^{13/19} \frac{1 - \Omega_*/\Omega_{\rm K}}{\eta^{17/19} R^{108/19}}, \tag{3}$$

where B_*, M_*, $\dot{M}$, Ω_*, $\Omega_{\rm K}$ and η represent the polar value of the poloidal magnetic field, the mass of the compact object, the mass accretion rate, the spin angular velocity, the Keplerian angular velocity and the diffusivity of the plasma respectively. It can be seen that for slow rotators material inside the corotation radius satisfies $\Omega_*/\Omega_{\rm K} = P_{\rm K}/P_* < 1$, resulting in an inward flow of material, while for regions in the accretion disc outside the corotation radius $\Omega_*/\Omega_{\rm K} = P_{\rm K}/P_* > 1$, resulting in an outward magneto-centrifugal push (Li & Wickramasinghe 1998) which effectively impedes the viscosity-driven radial inflow. The effectivity of the outward magneto-centrifugal push of disc material outside the corotation radius depends on the magnetospheric strength of the compact accreting object, and also on the diffusivity of the magnetospheric field of the compact object frozen into the disc plasma. It has been shown (Breedt 2005) (Fig 13.6, p.63) that this outward push in disc regions outside the corotation radius results in the inflow speed of a magnetized disc to be significantly less than for a non-magnetized Keplerian disc. It has been shown (Wang & Robertson 1984) that Kelvin-Helmholtz instabilities can be triggered in this outer disc regions where the rotational velocity of the magnetospheric field of the rotating object is faster than the Keplerian flow of the disc. This can result in magnetic bubbles that transfer angular momentum outward through the disc. It is believed that the net centrifugal effect of the faster magnetosphere anchored in the disc and the outward transported Kelvin-Helmholtz magnetized bubbles will result in an accumulation of the disc outside the corotation radius and an associated gradual increase in the disc surface density. The magnetization of the disc through magnetic bubbles and magnetospheric field that diffuse into the disc, will result in the conditions building up towards the trigger of a magneto-gravitational instability, which may trigger enhanced inward flow of the accumulated disc material and associated high mass accretion. The driving mechanism of this instability will be magnetic viscosity in the disc. A brief discussion of the basic principles of the magnetic viscosity in this context will now be presented.

3. Magnetic viscosity

It is believed that frozen-in magnetic fields will have a very significant influence on the dynamical properties of accretion disc plasma. It has been shown (Jackson 1975, Chapt. 10, pp. 475-479) that the flow of a conducting fluid across magnetic fields will result in a significantly modified flow profile. It has been shown (Jackson 1975) that for $M >> 1$ the fluid rams into a magnetic obstruction too strong to advect, and plasma has to trickle across the magnetic obstruction with the typical $\mathbf{E} \times \mathbf{B}$ drift velocity, while for $M << 1$ the magnetic field can be advected with the flow, which is a very effective and rapid transport process of magnetic flux over meaningful distance scales. This principle has been applied to model the mass transfer flow from magnetized secondary stars, for example in AE Aquarii (Meintjes 2004) and other magnetic cataclysmic variables (Meintjes & Jurua 2006). For example, Meintjes (2004) showed that for Roche lobe mass transfer across a strong magnetic field near the L_1 region, the flow will essentially be decelerated to such an extent that matter will build up behind the magnetic barrier leading to sporadic fragmented release of plasma and slingshot prominences. This may be the mechanism behind fragmented mass transfer in many cataclysmic variable systems.

It has been mentioned earlier that the interaction between the rapidly rotating magnetosphere and the slower disc outside the corotation radius of slow-rotators can result in the creation of magnetic bubbles that will transport angular momentum outwards. This will also result in the magnetization of the disc over length scales that comprise a significant fraction of the disc outside the corotation radius. This build-up phase from a situation where $M < 1$ (weakly magnetized state of the disc) to a situation where $M > 1$ (highly magnetized state of the disc) may define the timescale for the trigger of a magneto-gravitational instability in some classes of accretion driven systems. It will be shown that a magneto-gravitational instability, leading to rapid inflow through the disc and enhanced mass accretion, can be associated with the scenario where $M \geq 1$.

4. A magneto-gravitational instability and dwarf novae

The build-up of material in a magnetized disc may lead to the disc becoming more turbulent, since the flow outside the corotation radius has departed from a purely Keplerian profile (Breedt 2005). It can be shown that in the event of a turbulent disc, where the turbulent disc viscosity exceeds

$$\begin{aligned}\nu_{\rm T} &= (\mu_{\rm T}/\rho_{\rm disc}) \\ &= \alpha H_{\rm disc} c_s \\ &\geq 10^{13}\left(\frac{\alpha}{0.1}\right)\left(\frac{H_{\rm disc}}{10^8\,{\rm cm}}\right) \\ &\quad \left(\frac{c_{\rm s}}{10^6{\rm cm\ s^{-1}}}\right){\rm cm^2\ s^{-1}}\end{aligned}$$

and magnetic diffusivity $\eta_{\rm T} \sim (c^2/4\pi\sigma_{\rm T}) \geq 10^{13}{\rm cm^2\ s^{-1}}$ with frozen-in disc magnetic fields exceeding $B \sim 100$ G,

$$M > 1\left(\frac{B_{\rm disc}}{100\,{\rm G}}\right)\left(\frac{L}{H_{\rm disc}}\right)\left(\frac{\nu_{\rm T}}{10^{13}\,{\rm cm^2 s^{-1}}}\right)^{-1/2}\left(\frac{\sigma_{\rm T}}{10^7\,{\rm s^{-1}}}\right)^{1/2}. \quad (4)$$

With the disc magnetized to this level the gas will increasingly find the disc magnetic fields a barrier that will hamper the azimuthal flow. The braking effect of the azimuthal flow is illustrated by the equation of motion of the azimuthal flow cutting perpendicularly across magnetic fields frozen into the disc, which is

$$\rho\frac{d\mathbf{v}_\perp}{dt} = \mathbf{f}_\perp - \frac{\sigma_{\rm T}B^2}{c^2}(\mathbf{v}_\perp - \mathbf{w}), \quad (5)$$

where $\mathbf{f}_\perp$ represents the local non-electromagnetic forces influencing the gas flow in the disc and with $\mathbf{v}_\perp - \mathbf{w}$ representing the local relative velocity of the disc flow and the frozen-in magnetic fields. Notice that the drift velocity ($\mathbf{w}$) essentially determines the rate at which the magnetic field migrates through the fluid (Choudhuri 1998). The acceleration experienced by disc plasma will influence the local disc dynamics over an increment of time (δt), which is represented by

$$v_\phi = v_{\phi,i} + \frac{dv_\perp}{dt}\delta t = v_{\phi,i} + \left(\frac{\mathbf{f}_\perp}{\rho} - \frac{\sigma B^2 v_{\phi,i}}{\rho c^2}(1-\zeta)\right)\delta t, \quad (6)$$

with $\zeta = (w/v_{\phi,i}) < 1$. The viscous effect of the magnetic field on the azimuthal fluid flow manifests in the second term in the brackets above. It is clear that if the magnetic viscosity is substantial the azimuthal flow will be decelerated across the frozen-in disc fields. One can effectively show that the effective gravity for disc matter orbiting the compact star with mass (M_1) is

$$\mathbf{g}_{\rm eff,\perp} = \left(-\frac{GM_1}{R^2} + \frac{v_\phi^2}{R}\right)\hat{\mathbf{R}}. \quad (7)$$

From these equations $\mathbf{g}_{\rm eff} \to 0$ for Keplerian flow, i.e. $v_\phi \to v_{\rm K}$. However, one can see that if the flow is decelerated across the fields, $\mathbf{g}_{\rm eff,\perp} < 0$ ($v_\phi < v_{\rm K}$). This means that if the disc has reached critical levels of magnetization, for which the Hartmann number $M > 1$, the azimuthal flow will be decelerated such that the effective gravity will point radially inward, which implies that during these states the disc plasma will be advected inward. One can readily show that the effective gravity can be written as

$$\mathbf{g}_{\rm eff} = -\left(\frac{2GM_1}{R^2}\right)\left(\frac{\sigma B^2}{\rho c^2}\right)((1-\zeta)-\gamma))\,\delta t\hat{\hat{e}}_{\mathbf{R}}, \quad (8)$$

where $\gamma = ((\mathbf{f}_\perp/\rho v_{\rm K})/(\sigma B^2/\rho c^2)$. For a turbulent disc in this magnetized disc phase, using $\nu_{\rm T} = \mu_{\rm T}/\rho_{\rm disc}$ with ($\rho_{\rm disc} \sim 10^{-8}$ g cm^{-3}), one can show

$$|\mathbf{f}_\perp| = \mu_{\rm T}\nabla^2 v \approx 10^{-7}\left(\frac{\mu_{\rm T}}{10^5 {\rm g\ cm^{-1}s^{-1}}}\right)\left(\frac{v_\phi}{10^8\,{\rm cm\ s^{-1}}}\right)\left(\frac{L}{10^{10}\,{\rm cm}}\right)^{-2} {\rm g\ cm^{-2}s^{-2}}, \quad (9)$$

using $\gamma = (w/v_\phi) \sim 10^{-6}$, $\sigma_{\rm T} \sim 10^7\,{\rm s^{-1}}$ and $\rho_{\rm disc} \sim 10^{-8}\,{\rm g\ cm^{-3}}$. Therefore, the effective gravity when the magneto-gravitational instability is triggered is

$$\mathbf{g}_{\rm eff} = -3000\left(\frac{M_1}{M_\odot}\right)\left(\frac{R_{\rm disc}}{3\times10^{10}\,{\rm cm}}\right)^{-2}\left(\frac{\delta t}{1{\rm s}}\right)\hat{\mathbf{R}}\,{\rm cm\ s^{-2}}, \quad (10)$$

which is $\mathbf{g}_{\rm eff} = 3\mathbf{g}_{\rm earth}$ for the parameters chosen. This is a clear indication that magnetic viscosity in a magnetized disc will drive a significant mass inflow during events where the disc Hartmann number $M > 1$, which may be the situation during dwarf novae eruptions in cataclysmic variables. The net inward mass flow will then be the resultant effect of the inward advection of disc matter caused by the magneto-gravitational instability and the outward centrifugal effect caused by the faster rotating magnetosphere.

It has been mentioned earlier that turbulent viscosity has been parameterized in terms of a disc α-parameter, i.e. $\alpha_{\rm cold} \rightarrow 0.01$ for a cold disc and $\alpha_{\rm hot} \rightarrow (0.1 - 0.3)$ for a hot disc during a dwarf nova eruption. The model we propose quantifies the viscosity in terms of the disc Hartmann number, i.e. $M < 1$ for a cold disc and $M \geq 1$ when a large scale magneto-gravitational instability can be triggered, which will lead to enhanced inflow of disc material in the event of a dwarf nova eruption. It has been observed (Hellier 2001) that the most common outbursts are accompanied by an inward propagating heat-wave (h-w), that originates in the outer parts of the disc, which moves inward with a speed $v_{\rm h-w} \sim \alpha_{\rm hot} c_{\rm s}$, with $c_{\rm s} \sim 10^6 (T_{\rm disc}/10^4\,{\rm K})^{1/2}\,{\rm cm\,s^{-1}}$ the speed of sound in the hot state of the disc.

The model we propose above then can quantify the turbulent magnetic viscosity in terms of the state of magnetization of the disc, ($M_{\rm cold} < 1$; $M_{\rm hot} \geq 1$), which may be associated with turbulent viscosity, i.e. $\alpha_{\rm cold} \sim 0.01$; $\alpha_{\rm hot} \sim 0.1 - 0.3$. The fact that during the low state $M_{\rm cold} < 1$ has important consequences for the large-scale magnetic transport process and eventual total magnetization of the disc, just as $\alpha_{\rm cold} \sim 0.01$ results in the accumulation of disc material, which leads to the gradual increase of the disc surface density and resultant hot state which triggers the dwarf nova eruption.

5. Turbulent dissipation and heating

The trigger of a magneto-gravitational instability and associated large-scale inward advection of the azimuthal magnetized flow ($M_{\rm hot} \geq 1$) of the disc will be associated with the trigger of hydrodynamic turbulence as well and associated dissipation of turbulent mechanical energy. The rate of dissipation of mechanical energy per unit volume of the turbulent eddies (Choudhuri 1998, pp. 165-169) is

$$\dot{u}_{\rm T} = \rho_{\rm disc}\left(\frac{v^3_{\rm rms,d}}{l_{\rm d}}\right) = \rho_{\rm disc} v_{\rm T,d}\left(\frac{v_{\rm rms,d}}{l_{\rm d}}\right)^2, \qquad (11)$$

with $v_{\rm T,d} \approx v_{\rm rms,d} l_{\rm d}$, where $v_{\rm rms,d}$ represents the turn-over velocity of turbulent eddies in the dissipative regime, and $l_{\rm d}$ the smallest length scale of turbulent eddies in the dissipative regime. The dissipative length scales are of the order $l_{\rm d} = (R_{\rm e}/R_{\rm e,crit})^{-3/4} H_{\rm d}$ (Biskamp 2003), with $R_{\rm e}$ the Reynolds number of the flow, and $R_{\rm e,crit} \sim 1000$ the critical value for the trigger of turbulence in the flow (Tritton 1977). Here $H_{\rm disc}$ represents the maximum length scale of turbulent cells, which is constrained by the vertical disc height. For disc flow Reynolds numbers of the order $R_{\rm e} \sim 10^{14}$, and typical disc parameters, i.e. $H_{\rm disc} \sim 10^8$ cm, $\rho_{\rm disc} \sim 10^{-8}\,{\rm g\,cm^{-3}}$ and $v_{\rm rms} \leq c_{\rm s}$, one can show that $l_{\rm d} \leq 100$ cm and

$$\dot{u}_{\rm disc} \sim 10^8 \left(\frac{n_{\rm disc}}{6\times10^{15}\,{\rm cm^{-3}}}\right)\left(\frac{v_{\rm T,d}}{10^8\,{\rm cm^2 s^{-1}}}\right) \left(\frac{v_{\rm rms}}{c_{\rm s}}\right)^2 \left(\frac{l_{\rm d}}{100\,{\rm cm}}\right)^{-2} {\rm erg\,cm^{-3} s^{-1}}. \qquad (12)$$

The total disc luminosity in a fraction (β) of the disc volume during this "hot" stage can then be estimated as

$$L_{\rm disc} \approx \dot{u}_{\rm disc}(\beta R)^2 H_{\rm d} \leq 10^{35}\left(\frac{\dot{u}_{\rm disc}}{10^8\,{\rm erg\,cm^{-3}s^{-1}}}\right)\left(\frac{\beta}{0.1}\right)^2 \left(\frac{R_{\rm disc}}{3\times10^{10}\,{\rm cm}}\right)^2\left(\frac{H_{\rm disc}}{10^8\,{\rm cm}}\right) {\rm erg\,s^{-1}}. \qquad (13)$$

This upper limit is completely consistent with the observed luminosities of $L_{\rm disc} \sim 10^{34}\,{\rm erg\,s^{-1}}$ of systems like U Gem, SS Cyg, Z Cha and VW Hyi during outbursts, e.g. Warner (1995) and references therein.

6. Disc outflows

The draining of the accretion disc outside the corotation radius in the event of a magneto-gravitational instability, which could be associated with dwarf nova events, may result in the inward near-advection of the magnetospheric field anchored in the disc. This will lead to a deformation of the large scale magnetospheric field, with the associated outward directed centrifugal acceleration of plasma trapped in the disc corona (Blandford & Payne 1982). These authors have shown that outward centrifugal acceleration along the open field lines will be triggered for magnetospheric fields anchored in the disc with angles $i \leq 60°$. This may lead to significant centrifugally driven winds during these events. The observability of these centrifugally driven events may improve dramatically when powerful radio telescopes, like the SKA, become operational over the next decades.

7. Discussion

In this study the effect of frozen-in magnetic fields in the accretion disc has been investigated. It has been shown that the disc magnetization can be expressed in terms of the Hartmann number (M). It has been shown that the accretion discs in slow-rotators may cycle between states of low magnetization($M_{\rm cold} << 1$) and high magnetization ($M_{\rm hot} \geq 1$), in similar fashion as the disc cycling between states of low ($\alpha_{\rm cold} \sim 0.01$) and high ($\alpha_{\rm hot} \sim 0.1 - 0.3$) viscosity during "cold" and "hot" states. It has been shown that during low states of the disc, i.e. low magnetization ($M < 1$), corresponding to low viscosity ($\alpha \rightarrow \alpha_{\rm cold}$), the frozen-in magnetic field can readily be advected along with the flow, resulting in an effective magnetic field transport mechanism in the disc. It has been shown that in the cold state the centrifugal push of the magnetospheric fields anchored in the disc can result in a low inflow rate through the outer disc, interpreted as a low disc viscosity $\alpha \rightarrow \alpha_{\rm cold}$. This will result in a gradual build-up of disc material outside the corotation radius. As the disc grows, the magnetization will increase accordingly, which may lead to the trigger of a large scale magneto-gravitational instability when $M \geq 1$. This leads to an inward-driven mass flow since $\mathbf{g}_{\rm eff} < 0$, which will effectively drain the disc, which will be interpreted as a high effective viscosity $\alpha \rightarrow \alpha_{\rm hot}$. It has also been shown that dissipation of turbulent energy during this state can easily account for accretion disc luminosities $L_{\rm disc} \leq 10^{35}$ erg s^{-1}, which is sufficient to explain the energetics involved in all dwarf novae eruptions. The inward motion during the highly viscous state of the disc will result in magnetospheric fields anchored in the disc becoming highly distorted, which will lead to centrifugally accelerated outward winds from the disc. These centrifugally driven disc winds may become observable when sophisticated radio interferometers like the SKA become operational over the next decades.

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